1 volume conservation

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial x'} + \frac{\partial f}{\partial z'}\frac{\partial z'}{\partial x} \tag{1}$$
$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial y'} + \frac{\partial f}{\partial z'}\frac{\partial z'}{\partial y} \tag{2}$$
$$\frac{\partial f}{\partial z} = \frac{\partial f}{\partial z'}\frac{\partial z'}{\partial z} \tag{3}$$
$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial t'} + \frac{\partial f}{\partial z'}\frac{\partial z'}{\partial t} \tag{4}$$

for $f = z$, one obtains:

$$\frac{\partial z}{\partial x'} = -J\frac{\partial z'}{\partial x} \tag{5}$$
$$\frac{\partial z}{\partial y'} = -J\frac{\partial z'}{\partial y} \tag{6}$$
$$\frac{\partial z}{\partial t'} = -J\frac{\partial z'}{\partial t} \tag{7}$$

The volume conservations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{8}$$

$$\frac{\partial u}{\partial x'} + \frac{\partial u}{\partial z'}\frac{\partial z'}{\partial x} + \frac{\partial v}{\partial y'} + \frac{\partial v}{\partial z'}\frac{\partial z'}{\partial y} + \frac{\partial w}{\partial z'}\frac{\partial z'}{\partial z} = 0 \tag{9}$$

$$J\frac{\partial u}{\partial x'} + J\frac{\partial z'}{\partial x}\frac{\partial u}{\partial z'} + J\frac{\partial v}{\partial y'} + J\frac{\partial z'}{\partial y}\frac{\partial v}{\partial z'} + \frac{\partial w}{\partial z'} = 0 \tag{10}$$

$$J\frac{\partial u}{\partial x'} - \frac{\partial z}{\partial x'}\frac{\partial u}{\partial z'} + J\frac{\partial v}{\partial y'} - \frac{\partial z}{\partial y'}\frac{\partial v}{\partial z'} + \frac{\partial w}{\partial z'} = 0 \tag{11}$$

since

$$\frac{\partial z}{\partial x'}\frac{\partial u}{\partial z'} = \frac{\partial}{\partial z'}\left(\frac{\partial z}{\partial x'}u\right) - \frac{\partial J}{\partial x'}u \tag{12}$$
$$\frac{\partial z}{\partial y'}\frac{\partial v}{\partial z'} = \frac{\partial}{\partial z'}\left(\frac{\partial z}{\partial y'}v\right) - \frac{\partial J}{\partial y'}v \tag{13}$$

$$J\frac{\partial u}{\partial x'} + \frac{\partial J}{\partial x'}u + J\frac{\partial v}{\partial y'} + \frac{\partial J}{\partial y'}v + \frac{\partial w}{\partial z'} - \frac{\partial}{\partial z'}\left(u\frac{\partial z}{\partial x'}\right) - \frac{\partial}{\partial z'}\left(v\frac{\partial z}{\partial y'}\right) = 0 \tag{14}$$

$$\frac{\partial Ju}{\partial x'} + \frac{\partial Jv}{\partial y'} + \frac{\partial}{\partial z'}\left(w - u\frac{\partial z}{\partial x'} - v\frac{\partial z}{\partial y'}\right) = 0 \tag{15}$$

The definition of ω

$$\omega = \frac{\partial z'}{\partial t} + u\frac{\partial z'}{\partial x} + v\frac{\partial z'}{\partial y} + w\frac{\partial z'}{\partial z} \tag{16}$$

$$\begin{aligned} w &= J\omega - J\frac{\partial z'}{\partial t} - Ju\frac{\partial z'}{\partial x} - Jv\frac{\partial z'}{\partial y} & (17) \\ &= J\omega + \frac{\partial z}{\partial t'} + u\frac{\partial z}{\partial x'} + v\frac{\partial z}{\partial y'} & (18) \end{aligned}$$

subsituting w

$$\frac{\partial Ju}{\partial x'} + \frac{\partial Jv}{\partial y'} + \frac{\partial}{\partial z'}\left(J\omega + \frac{\partial z}{\partial t'}\right) = 0 \tag{19}$$

finally:

$$\frac{\partial J}{\partial t'} + \frac{\partial Ju}{\partial x'} + \frac{\partial Jv}{\partial y'} + \frac{\partial J\omega}{\partial z'} = 0 \tag{20}$$