EWPF with non-linear $\mathcal{H}$

Sanita Vetra-Carvalho   Peter Jan van Leeuwen

University of Reading

s.vetra-carvalho@reading.ac.uk

2nd April 2014

In between observations

Advance each particle in time using proposal density

$$\mathbf{x}_j^n = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + B(\tau)\left(\mathbf{y}^{(k)} - \mathcal{H}\left(\mathbf{x}_j^{(m-1)}\right)\right) + d\beta_j^n$$

Accumulate weights

$$w_j^n = w_j^{n-1}\frac{p(\mathbf{x}^n|\mathbf{x}^{n-1})}{q(\mathbf{x}^n|\mathbf{x}^{n-1},\mathbf{y}^k)}$$

At observation time

- Find max weight for each particle using limited-memory BFGS method

$$C_j = -w_j^{rest} + \left(\mathbf{x}_j^k - \mathcal{M}(\mathbf{x}_j^{k-1})\right)^T \mathbf{Q}^{-1} \left(\mathbf{x}_j^k - \mathcal{M}(\mathbf{x}_j^{n-1})\right) + \left(\mathbf{y}_j^k - \mathcal{H}(\mathbf{x}_j^k)\right)^T \mathbf{R}^{-1} \left(\mathbf{y}_j^k - \mathcal{H}(\mathbf{x}_j^k)\right) o$$

- Keep particle if $C_j > C_{max}$
- Find α (using Newton/bisection) such that $C_{max} - C_j = 0$ giving

$$C_{max} = -w_j^{rest} + \left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{k-1})\right)^T \mathbf{Q}^{-1} \left(\mathbf{x}_j^* - \mathcal{M}(\mathbf{x}_j^{n-1})\right) + \left(\mathbf{y}_j^k - \mathcal{H}(\mathbf{x}_j^*)\right)^T \mathbf{R}^{-1} \left(\mathbf{y}_j^k - \mathcal{H}(\mathbf{x}_j^*)\right)$$

$$\mathbf{x}_j^* = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + \alpha_j \mathbf{K}\left(\mathbf{y} - \mathcal{H}\left(\mathcal{M}\left(\mathbf{x}_j^{n-1}\right)\right)\right)$$

Lorenz63 model

$$
\begin{aligned}
x^{n+1} &= x^n + \Delta t \sigma (y^n - x^n) + \xi_x^n \\
y^{n+1} &= y^n + \Delta t (\rho x^n - y^n - x^n z^n) + \xi_y^n \\
z^{n+1} &= z^n + \Delta t (x^n y^n - \beta^n z^n) + \xi_z^n
\end{aligned}
$$

Settings:

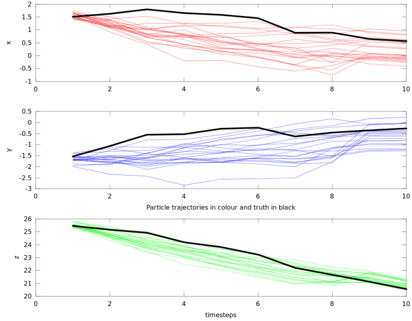
- $dt = 0.01$
- $\mathbf{x}^0 = [1.508870, -1.531271, 25.4609]$
- $\sigma = 10,\ \rho = 28,\ \beta = 8/3$
- Stand. dev. of initial ensemble and observational error is $\sigma_R = \sqrt{2}$.
- Stand. dev. of model error is $\sigma_Q = \sqrt{2}\Delta t$.

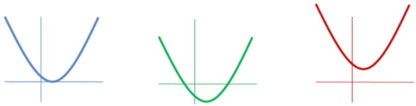
Issues with $\mathcal{H}$

- Choice of observations ($\mathbf{y} = \mathcal{H}(xy)$, $\mathcal{H}(yz)$)
- Choice of nudging term in

$$\mathbf{x}_j^n = \mathcal{M}\left(\mathbf{x}_j^{n-1}\right) + B(\tau)\left(\mathbf{y}^{(k)} - \mathcal{H}\left(\mathbf{x}_j^{(m-1)}\right)\right) + d\beta_j^n$$

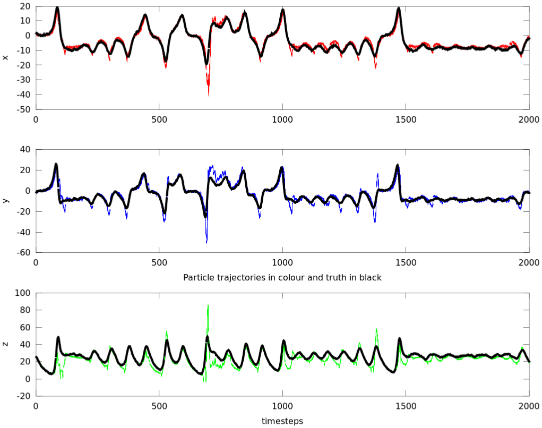
- Choice of alphas
- What to do if not enough particles can reach C_{max}.





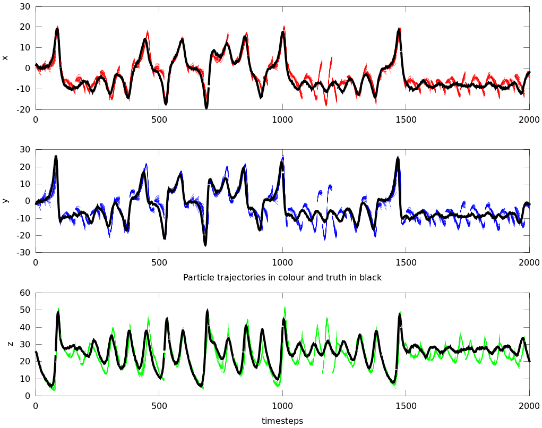
$\mathcal{H}(\mathbf{x}) = yz$

$\Delta t_{obs} = 10$



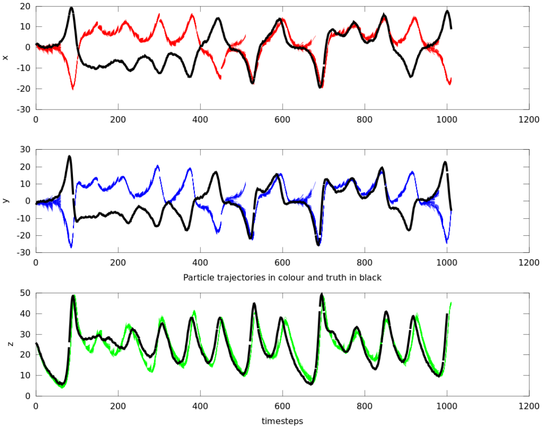
$\mathcal{H}(\mathbf{x}) = yz$

$\Delta t_{obs} = 20$



$\mathcal{H}(\mathbf{x}) = xy$

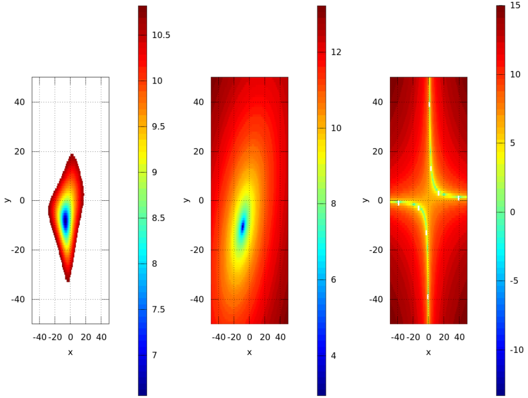
$\Delta t_{obs} = 10$



$\mathcal{H}(\mathbf{x}) = xy$

$\Delta t_{obs} = 10$

$$C_j = -w_j^{rest} + \left(\mathbf{x}_j^k - \mathcal{M}(\mathbf{x}_j^{k-1})\right)^T \mathbf{Q}^{-1} \left(\mathbf{x}_j^k - \mathcal{M}(\mathbf{x}_j^{n-1})\right) + \left(\mathbf{y}_j^k - \mathcal{H}(\mathbf{x}_j^k)\right)^T \mathbf{R}^{-1} \left(\mathbf{y}_j^k - \mathcal{H}(\mathbf{x}_j^k)\right)$$



$\mathcal{H}(\mathbf{x}) = xy$

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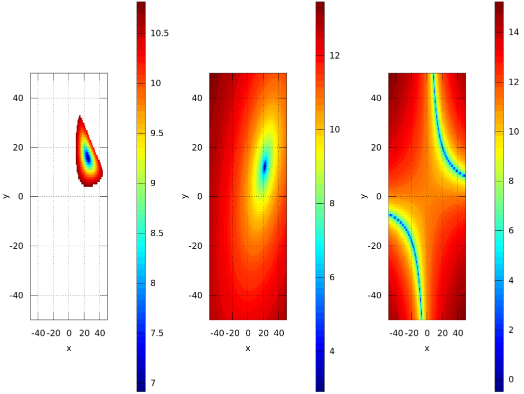

THANK YOU!

ANY QUESTIONS?