Aspects of localization and smoothing in ensemble Kalman filters

Lars Nerger and Paul Kirchgessner

Alfred Wegener Institute for Polar and Marine Research
Bremerhaven, Germany

with contributions from

Svenja Schulte and Angelika Bunse-Gerstner

University of Bremen

SANGOMA

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Outline

2 parts:

- Lars:

  Effect of nonlinearity on ensemble smoothing

- Paul:

  What makes on optimal localization radius?

Effect of nonlinearity on ensemble smoothing

Ensemble smoothing

- Very simple (ensemble array $\mathbf{X}^f_{k|k-1}$)

  Filter: $$\mathbf{X}^a_{k|k} = \mathbf{X}^f_{k|k-1}\mathbf{G}_k$$

  Smoother: $$\mathbf{X}^a_{k-1|k} = \mathbf{X}^a_{k-1|k-1}\mathbf{G}_k$$

(Discussed, e.g. by Cosme et al., MWR 2012)

- Optimal for linear systems:
  - ➜ Forecast of smoothed state = analysis at later time
- No longer true for nonlinear systems!

➜ What is the effect of the nonlinearity?

➜ Do ensembles just decorrelate? (See Cosme et al. 2010)

Numerical study with Lorenz-96

- Cheap and small model (one of our benchmarks)
- Global and local filters possible
- Nonlinearity controlled by forcing parameter *F*
  - periodic waves up to F=4
  - non-periodic for F>4

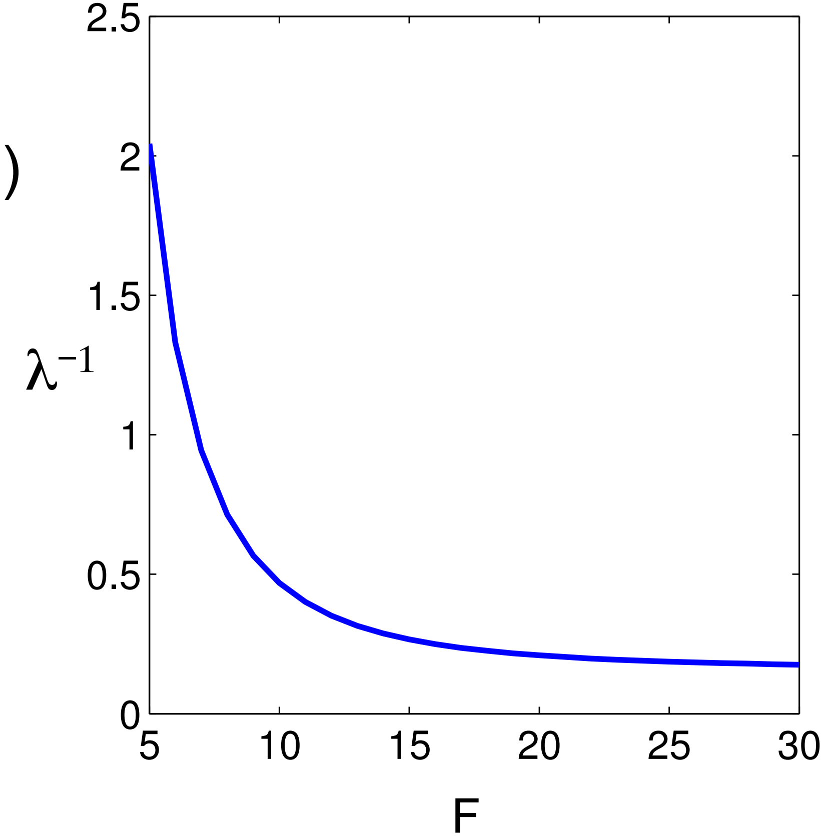
- Experiments over 20000 time steps
- Vary F, smoother lag, and forecast length
- Consider mean RMS errors
- Tune inflation for minimal RMS errors

Influence of forcing on nonlinearity

Indicator of nonlinearity:

- Lyapunov time λ^{-1}
  (errors grow asymptotically by *e*)

- Related to error-doubling time

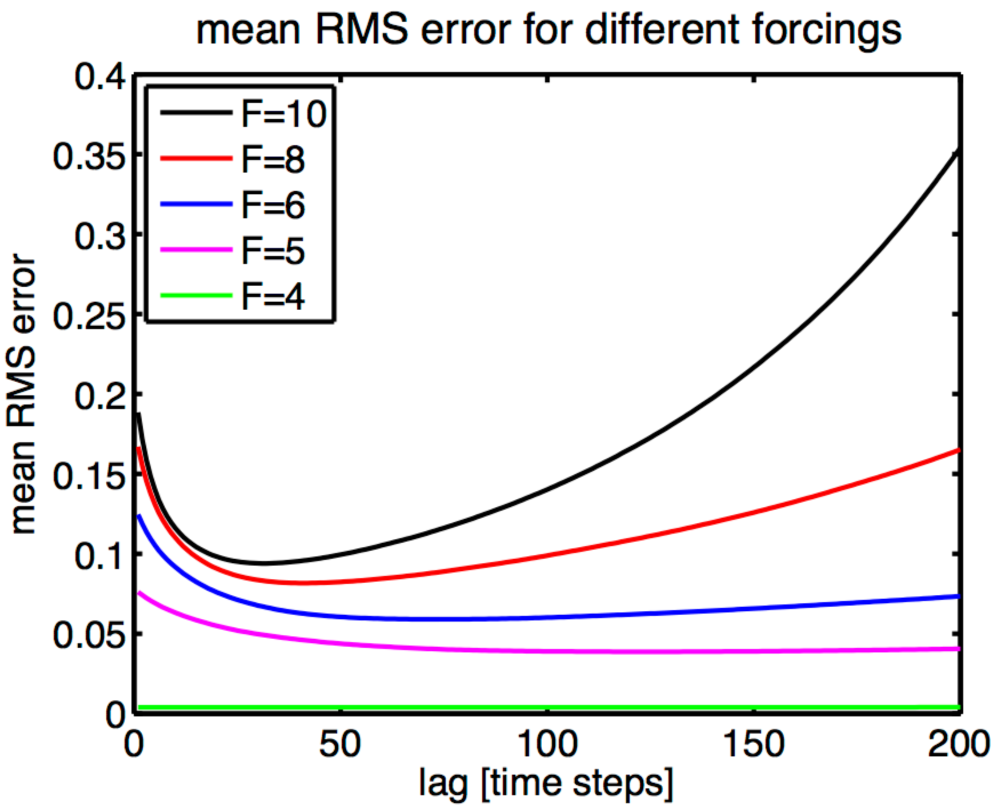


Source: A. Karimi, M.R. Paul, Chaos 20, 043105 (2010)

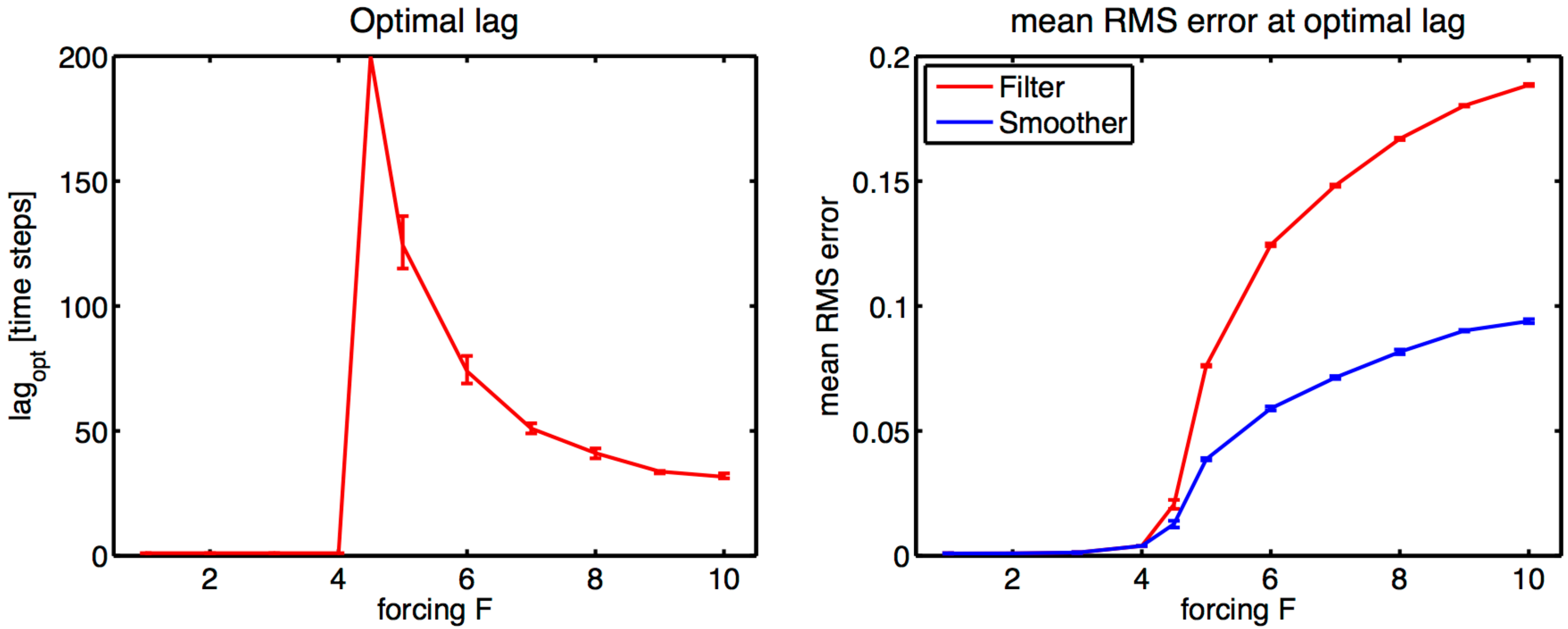
Optimal lag

- Assimilate at each time step
- ensemble size 34, global ESTKF

- Very small RMS up to F=4
- Strong growth in RMS for F>4
- Clear impact of smoother
- RMS errors grow beyond an „optimal“ lag

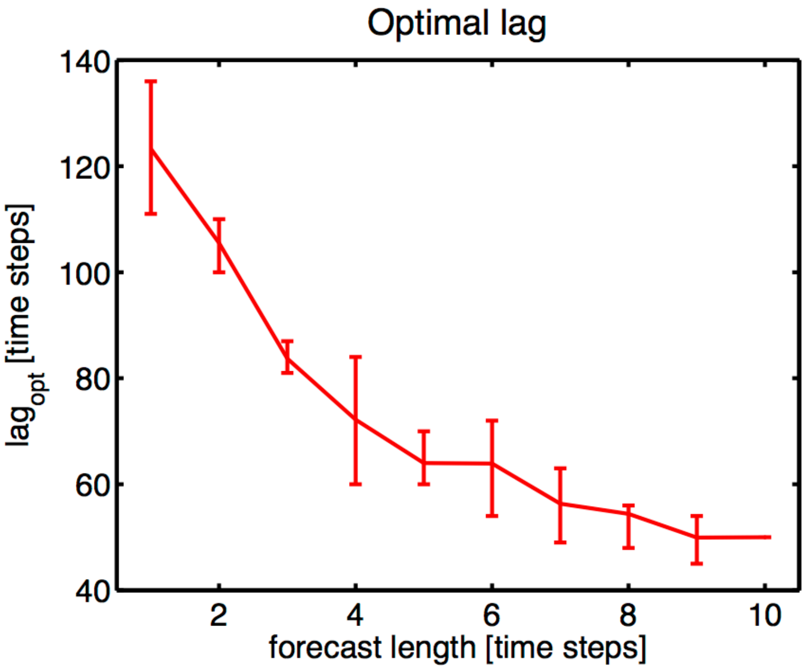


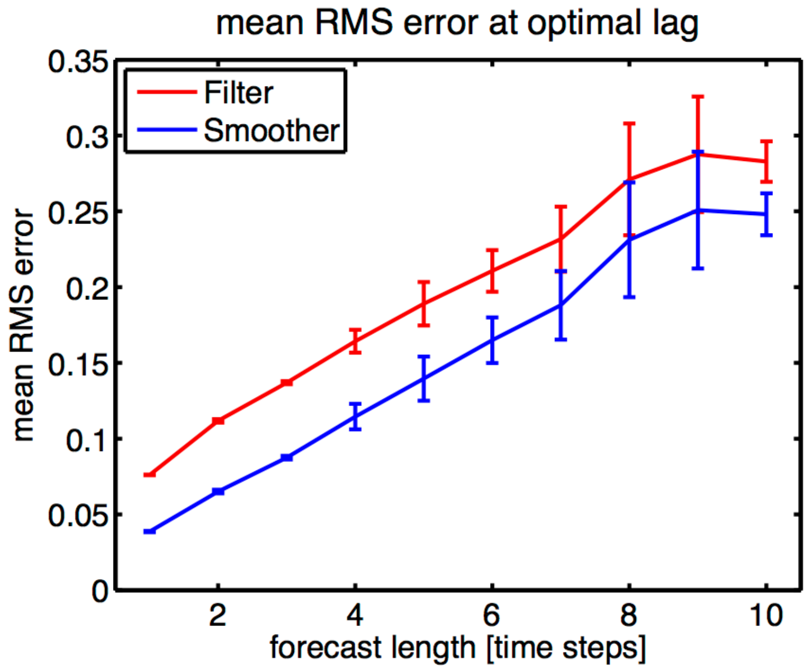
Impact of smoothing



- Optimal lag (minimal RMS error)
  - Behavior similar to error-doubling time
- RMS error at optimal lag
  - Smoother reduces error by 50% for all F>4

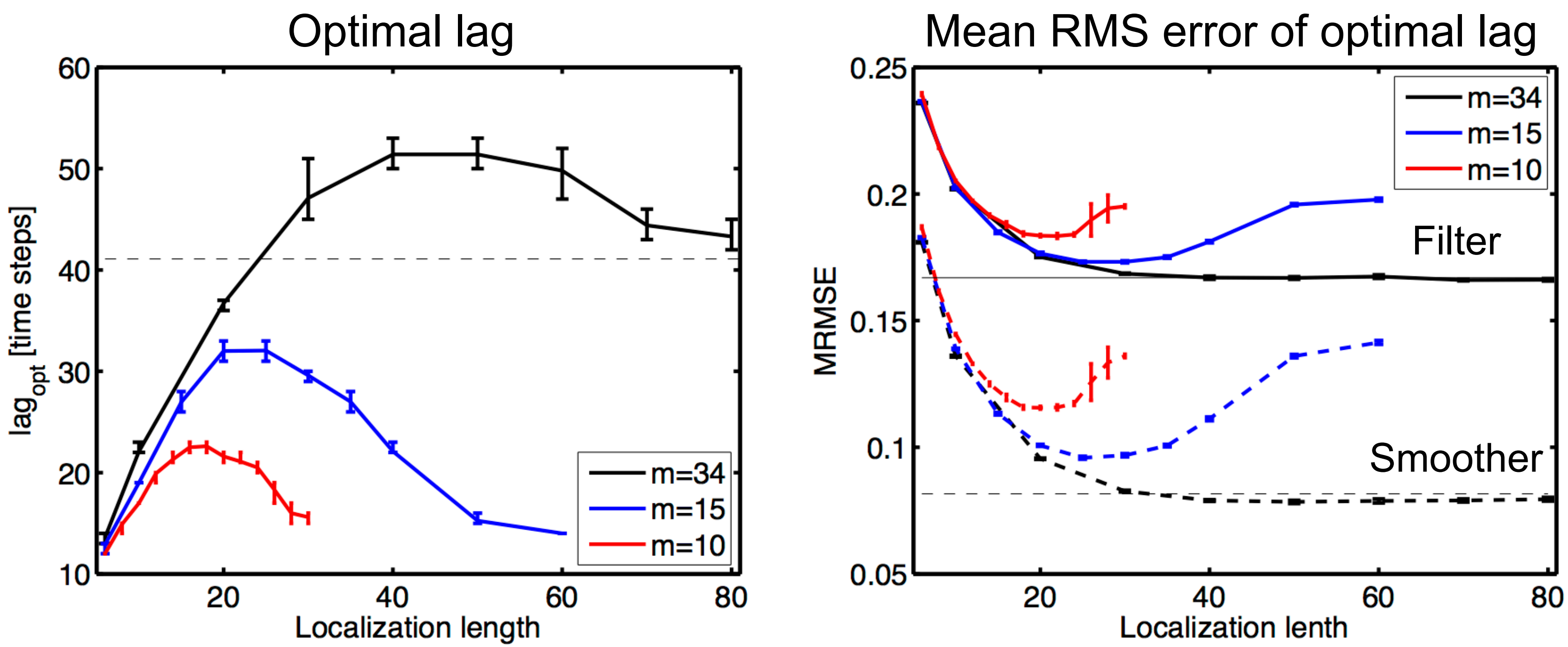
Vary forecast length (F=5)





- Forecast length = time steps over which nonlinearity acts on ensemble
- Optimal lag shrinks, then stagnates
- RMS errors grow for filter and smoother
  - Improvement by smoother is constant

Influence of Localization on Smoothing



- Small improvement of smoothing with localization for m=34
- Shorter optimal lag for smaller m
- Smoother profits more from larger m

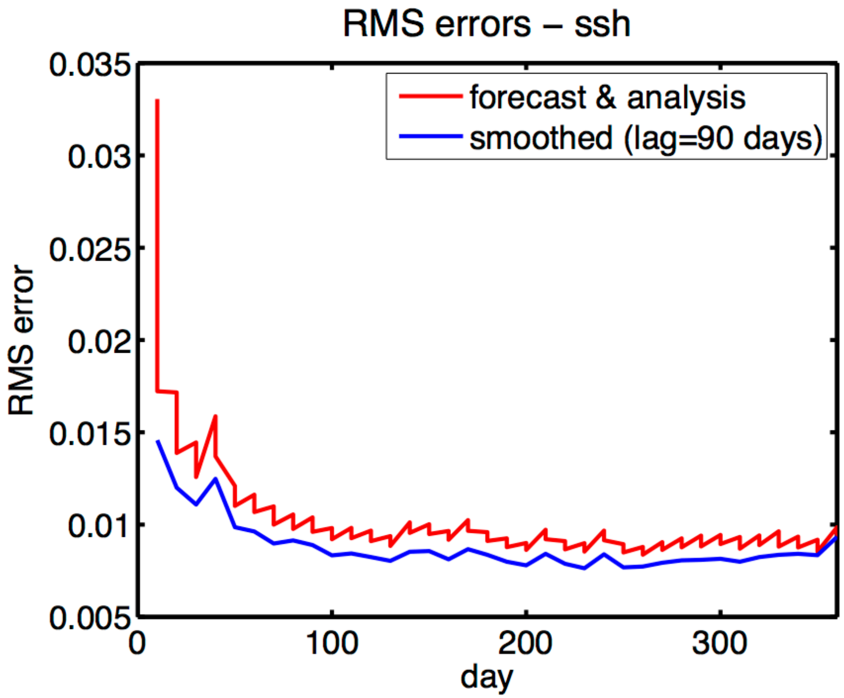
➜ Related to sampling quality for different m

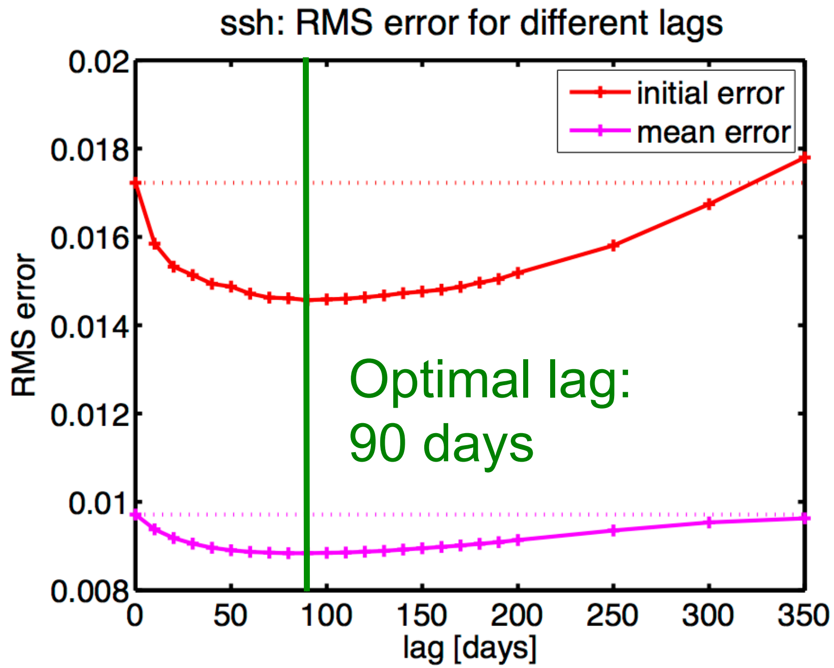
Smoothing with global ocean model

- FESOM (Finite Element Sea-ice Ocean model)
- Global configuration
  - 1.3° resolution, 40 levels
  - Horizontal refinement at equator
  - State vector size 10^7

- Twin experiment for SSH
  - Ensemble size 16
  - Assimilate each 10th day
  - ESTKF with smoother extension
    (Same code as for Lorenz-96)
  - Inflation tuned for optimal performance (ρ=0.9)

Effect of smoothing on global model





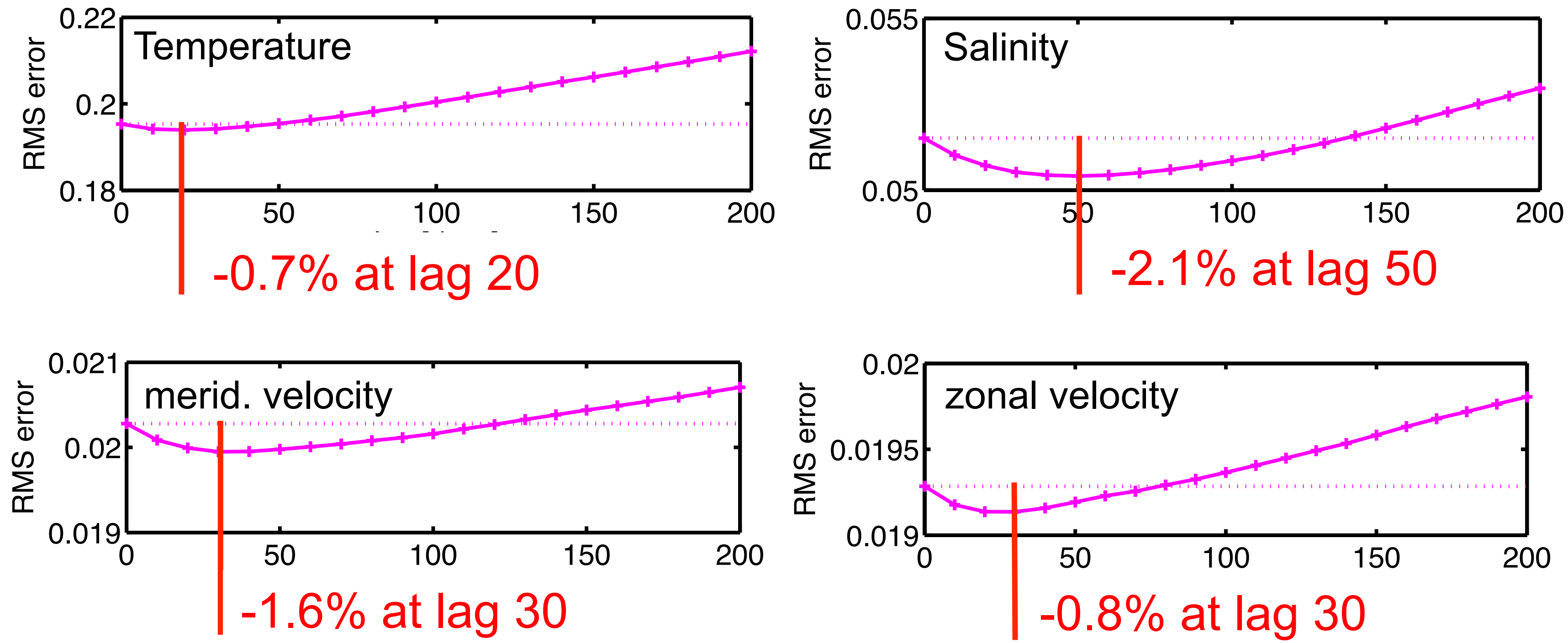
RMSe reduced by smoother:

- ~15% at initial time
- ~8% over the year

(Not new! See e.g. Cosme et al. 2010)

- Large impact of each lag up to 50 days
- Optimal lag about 90 days
- Deterioration for very long lag (not only effect for inflation!)

Multivariate effect of smoothing



- Multivariate impact smaller & specific for each field
- Optimal lag dependent on field
- Optimal lag smaller than for SSH

➜ What is the optimal lag for multivariate assimilation?

Summary

- Optimal ensemble smoothing with linear models
- Nonlinearity influences ensemble smoothers
  - Optimal smoothing lag
  - Same inflation for filter and smoother
  - Difference in nonlinearity of dynamics and due to forecast length
- Influence of localization
  - Optimal smoothing coincides with optimal localization length of filter (max. optimal lag)
- Varying optimal smoother lag in multivariate assimilation
  - Shorter optimal multivariate lags